

Relativistic interaction of rippled laser beams with plasmas

M.V. ASTHANA, A. GIULIETTI, D. GIULIETTI, L.A. GIZZI, AND M.S. SODHA

Istituto di Fisica Atomica e Molecolare, Area della Ricerca-CNR, Via Alfieri, 1, Ghezzano, 56010, Pisa, Italy

(RECEIVED 17 August 1999; ACCEPTED 14 April 2000)

Abstract

An investigation of the growth of a radially symmetrical ripple, superimposed on a Gaussian laser beam in a plasma is presented. Based on WKB and paraxial ray approximation the phenomenon of relativistic self-focusing (RSF) is analytically investigated. The differential equation for beamwidth parameter of rippled laser beam is evaluated. The ripple gets focused when the initial power of the ripple is greater than the critical power for focusing. The focusing is found to be considerably affected by the power of the main beam and the phase angle between the electric vectors of the main beam and the ripple. At higher intensities the saturation effects of nonlinearity become predominant, making the nonlinear refractive index in the paraxial region have slower radial dependence, and thus the ripple extract relatively less energy from its neighborhood. The case of magnetized plasmas is also preliminarily discussed.

1. INTRODUCTION

Currently there is much interest in the interaction of high intensity ultra-short laser pulses with plasmas, both from basic research point of view and potential applications in advanced fusion energy, X-ray lasers and ultrahigh-gradient electron accelerators (Tajima & Dawson, 1979; Tabak *et al.*, 1994). New short pulse laser technology has made possible the production of extremely intense laser sources at the multi-terawatt level. The focused intensities obtained are very high $\approx 10^{18}$ W/cm² (Patterson *et al.*, 1991; Main & Mourou, 1988), and further developments are aimed at intensities exceeding 10^{21} W/cm². It is hereby now possible to explore relativistic laser-plasma interaction.

One of the most promising topics of investigation is relativistic self-focusing (RSF) and self-channeling of an intense electromagnetic wave in a plasma. Self-focusing results from an increase of the on-axis index of refraction relative to the edge of the laser beam through depressing the on-axis electron density (ponderomotive self-focusing) or through the radially dependent relativistic correction to the plasma frequency when the quiver velocity of the electrons in the laser field becomes relativistic (RSF), or both. The analysis presented here will be concerned only with RSF on a time scale sufficiently short such that the plasma density profile

does not evolve sufficiently under the influence of the laser beam. This implies that the pulse length τ_R of the laser beam must be short compared with τ_s ($= r/c_s$), the time scale for the density depression to occur (where r is the radii of the beam and c_s is the ion-sound speed). An analysis in terms of the envelope and paraxial approximations shows that, depending on laser pulse and plasma parameters, either self-focusing of the whole pulse or pulse filamentation occurs (Asthana *et al.*, 1994; Borisov *et al.*, 1995).

The origin of filamentation instability may be attributed to small scale density perturbations (resulting from quasi-neutrality) or small scale intensity spikes associated with the main beam. The perturbations grow at the cost of the main beam and this is detrimental to the cause of laser-induced fusion. Thus, direct and indirect experimental evidence reveals that an apparently smooth looking laser beam has intensity spikes that may lead to distortion of self-focusing in nonlinear media (Kothari & Kobayashi, 1983; Pandey *et al.*, 1990). In view of ongoing ultra short pulse layers, we make a comprehensive analysis on relativistic interaction of rippled laser beams with plasmas. Foremost in Section 2, the nonlinear dielectric constant due to relativistic variation of mass is presented. Next, the RSF equation of the rippled laser beam is evaluated under WKB and paraxial ray theory. Further, analytically the expression for the growth of ripple is presented followed by the effect of static magnetic field. Section 3 deals with results and discussions along with scope of future work.

Address correspondence and reprint requests to: A. Giulietti, Istituto di Fisica e Molecolare, Area della Ricerca-CNR, Via Alfieri, 1, Ghezzano, 56010, Pisa, Italy. E-mail: tonino@ifam.pi.cnr.it

2. NONLINEAR DIELECTRIC CONSTANT OF THE PLASMA

The relativistic equation of motion for a point charge is

$$\frac{d}{dt} \left[\frac{m_o v}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \right] = e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (1)$$

where

m_o = the rest mass of a point charge

c = velocity of light in vacuum

\mathbf{E} = electric field

\mathbf{B} = wave magnetic field.

For a homogeneous plane wave traveling in the direction of a unit vector \hat{n} , the electric and magnetic vectors are perpendicular to \hat{n} and are related by $B = [\hat{n} \times (E/c)]$. Substituting for \mathbf{B} on the right hand side of the equation of motion (1) gives

$$\frac{d}{dt} \left[\frac{\mathbf{v}}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \right] = \frac{e}{m_o} \left(1 - \frac{\hat{n} \cdot \mathbf{v}}{c}\right) \mathbf{E} + \frac{e}{m_o c} (\mathbf{E} \cdot \mathbf{v}) \hat{n}. \quad (2)$$

Taking scalar product of Eq. (2) with \hat{n} , together with the energy equation establishes that,

$$\lambda \equiv \left[\frac{1 - \frac{\hat{n} \cdot \mathbf{v}}{c}}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \right] \text{ is constant.}$$

We now eliminate t in favor of the phase variable

$$\phi = \omega \left(t - \frac{\hat{n} \cdot \mathbf{r}}{c} \right), \quad (3)$$

where ω is the angular frequency of the wave and \mathbf{r} is the position vector of the particle. Then

$$v = \frac{dr}{dt} = \omega \left(1 - \frac{\hat{n} \cdot \mathbf{v}}{c}\right) r', \quad (4)$$

where the prime denotes the differentiation with respect to ϕ . Eliminating above equations we get

$$\frac{d}{dt} \left[\frac{\mathbf{v}}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \right] = \lambda \frac{d}{dt} \left(\frac{\mathbf{v}}{1 - \frac{\hat{n} \cdot \mathbf{v}}{c}} \right) = \omega^2 \lambda \left(1 - \frac{\hat{n} \cdot \mathbf{v}}{c}\right) \mathbf{r}''$$

and

$$\mathbf{r}'' = \frac{e}{\lambda m_o \omega^2} \left[\mathbf{E} + \frac{m}{c} (\mathbf{E} \cdot \mathbf{r}') \hat{n} \right], \quad (5)$$

which is second order differential equation for \mathbf{r} as a function of ϕ . The dielectric constant of the plasma is given by

$$\varepsilon = \left(1 - \frac{\omega_p^2}{\omega^2} \frac{m_o}{m_r}\right) \quad (6)$$

where

$m_r = \gamma m_o =$ relativistic mass

$\gamma = [1 - (v^2/c^2)]^{-1/2} =$ relativistic factor

$\omega_p = [(4\pi n e^2/m_o)]^{1/2} =$ plasma frequency in the absence of the beam.

The dielectric constant at arbitrary large nonlinearity, for a circularly polarized wave can be written as

$$\varepsilon = \varepsilon_o + \phi(E_o E_o^*) \quad (7)$$

where:

$\varepsilon_o = [1 - (\omega_p^2/\omega^2)]$ is the linear part of dielectric constant

and

$\phi(E_o E_o^*) = \frac{\omega_{po}^2}{\omega^2} \left[1 - \left(1 + \frac{\alpha E_o E_o^*}{2} C_1\right)^{-1/2} \right]$ is the nonlinear relativistic term,

$$\text{here } \alpha = \left(\frac{2e^2}{m_o^2 \omega^2 c^2} \right); C_1 = \left(1 + \frac{e^2 E^2}{16\lambda^2 m_o^2 \omega^2 c^2} \right). \quad (8)$$

2.1. Self-focusing equation of rippled laser beam

Consider the propagation of a Gaussian laser beam with a ring ripple superimposed on it in homogeneous collisionless plasma along the z -direction. The electric field at the fixed plane $z = 0$ of the main beam may be represented by

$$\mathbf{E}_o(z = 0) = \mathbf{E}_{o0} \exp\left(-\frac{r^2}{2r_o^2}\right) \exp(i\omega t), \quad (9)$$

where ω is the angular frequency of the laser beam, r is the radial coordinate of the cylindrical coordinate system and r_o is the initial width of the main beam. The electric field of the ring superimposed on the main beam may be expressed as

$$\mathbf{E}_1(z = 0) = \mathbf{E}_{10} \left(\frac{r}{r_{10}}\right) \exp\left(-\frac{r^2}{2r_1^2}\right) \exp(i\omega t - \phi_p), \quad (10)$$

with ϕ_p is the phase difference in between the main beam and the ripple; r_1 is the width of the ripple, with the maximum field of the ripple at $r = r_{10}$. The total electric vector of the beam can thus be written as $\mathbf{E} = \mathbf{E}_o + \mathbf{E}_1$. The intensity distribution of the rippled Gaussian laser beam is thus given by

$$\begin{aligned}
 EE^*(z=0) &= E_{00}^2 \exp\left(-\frac{r^2}{r_o^2}\right) \\
 &\times \left\{ 1 + 2 \frac{E_{10}}{E_{00}} \frac{r}{r_{10}} \cos \phi_p \exp \frac{r^2}{2} \times \left(\frac{1}{r_o^2} - \frac{1}{r_1^2} \right) \right. \\
 &\quad \left. + \frac{E_{10}^2}{E_{00}^2} \left(\frac{r}{r_{10}} \right)^2 \exp \left[r^2 \left(\frac{1}{r_o^2} - \frac{1}{r_1^2} \right) \right] \right\} \quad (11)
 \end{aligned}$$

The wave equation governing the electric vector of beam in plasma with dielectric constant ε can be written as

$$\nabla^2 \mathbf{E} + \frac{\omega^2}{c^2} \varepsilon \mathbf{E} = 0 \quad (12)$$

In writing Eq. 8, the term $\nabla(\nabla \cdot \mathbf{E})$ was neglected which is justified when $[(c^2/\omega^2)|(1/\varepsilon)\nabla^2 \ln \varepsilon] \ll 1$. Using WKB approximation and following Sodha *et al.* (1976), one can write

$$E(r, z) = A(r, z) \left[\frac{k(0)}{k(f)} \right]^{1/2} \exp[-i \int k(f) dz]$$

$$\text{where } k(f) = \left(\frac{\omega}{c} \right) [\varepsilon_o'(f)]^{1/2}, k(0) = \left(\frac{\omega}{c} \right) [\varepsilon_o'(f=1)]^{1/2} \quad (13)$$

Substituting for E and ε in Eq. (12), one obtains

$$-2ik(f) \left(\frac{\partial A}{\partial z} \right) + \nabla^2 A + \left(\frac{\omega^2}{c^2} \right) \psi(f) A = 0 \quad (14)$$

Putting $A(r, z) = A_o(r, z) \exp[-i \int k(f) dS]$ and separating real and imaginary parts, we find

$$2 \left(\frac{\partial S}{\partial z} \right) + \left(\frac{\partial S}{\partial r} \right)^2 + \frac{\omega^2}{c^2} \left[\frac{\varepsilon_1(f)}{k^2(f)} \right] = \frac{1}{k^2 A_o} \left[\frac{\partial^2 A}{\partial r^2} + \frac{1}{r} \frac{\partial A_o}{\partial r} \right]$$

and

$$\frac{\partial A_o^2}{\partial z} + \frac{\partial S}{\partial r} \left(\frac{\partial A_o^2}{\partial r} \right) + A_o^2 \left[\frac{\partial^2 S}{\partial r^2} + \frac{1}{r} \left(\frac{\partial S}{\partial r} \right) \right] = 0. \quad (15)$$

Eliminating the above equations and using the paraxial ray approximation, we obtain the RSF equation as (Asthana *et al.*, 1999)

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2(f) r_o^4 f^3} \left[1 - \left(\frac{\omega_p r_o}{c} \right)^2 \frac{f^2}{2} \frac{K \left[1 + \frac{KX}{4\lambda^2} \right]}{\left[1 + KX \left(1 + \frac{KX}{16\lambda^2} \right) \right]^{3/2}} XY \right]$$

$$\text{with } K = \frac{k(0)}{k(f)} \frac{\alpha E_{00}^2}{2f^2}, X = \left[1 + \frac{E_{10} r_o}{E_{00} r_{10}} \cos \phi_p \right] \quad (16)$$

and

$$Y = \left[1 - \left(\frac{E_{10} r_o}{E_{00} r_{10}} \right) \cos \phi_p \left(1 - \frac{1}{2} \left(\frac{r_o^2}{r_{10}} + 1 \right) \right) - \left(\frac{E_{10} r_o}{E_{00} r_{10}} \right)^2 \right].$$

For an initial Gaussian plane wave front of the beam, the initial conditions are $f(z=0) = 1$ and $(df/dz)|_{z=0} = 0$. When the two terms on the right hand side of Eq. (16) cancel each other at $z=0$, $[(d^2 f/dz^2)] = 0$. If one further considers a parallel beam at $z=0$ then $(df/dz) = 0$ at $z=0$; thus, if $f=1$ at $z=0$, it remains so for all values of z (in other words, the beam propagates without convergence or divergence). The self-trapping condition therefore is

$$\left[\frac{\omega_p r_o}{c} \right]^2 = \frac{2 \left[1 + \frac{\alpha E_{00cr}^2}{2} X \left(1 + \frac{1}{16\lambda^2} \frac{\alpha E_{00cr}^2}{2} X \right) \right]^{3/2}}{\left[1 + \frac{1}{4\lambda^2} \frac{\alpha E_{00cr}^2}{2} X \right] \frac{\alpha E_{00cr}^2}{2} XY} \quad (17)$$

The corresponding critical beam power is

$$\begin{aligned}
 P_{cr} &= \frac{c}{8\pi} \int \varepsilon^{1/2} E_{00cr}^2 2\pi r dr \\
 &= \frac{c}{8} r_o^2 E_{00cr}^2 \left[1 - \frac{\omega_p^2}{\omega^2} \frac{1}{\left[1 + \frac{\alpha E_{00cr}^2}{2} X \left(1 + \frac{1}{16\lambda^2} \frac{\alpha E_{00cr}^2}{2} X \right) \right]^{1/2}} \right] W. \quad (18)
 \end{aligned}$$

2.2. Growth of ripple

In the previous section we have written an equation for RSF of a rippled Gaussian laser beam in plasmas. The saturating nature of nonlinearity limits the radial dependence and energy transfer between the main beam and the ripple. This is further analyzed in the present section. Consider the propagation of a cylindrically symmetric Gaussian laser beam in a uniform, static and collisionless plasma:

$$\mathbf{E} = \mathbf{A}(r, z) e^{i(\omega t - kz)}$$

$$|A|_{2z} = A_{oo} e^{-r^2/r_o^2}; k = \frac{\omega}{c} (1 - \omega_{po}^2/\omega^2)^{1/2}, \quad (19)$$

where ω_{po} is the unperturbed plasma frequency. The behavior of complex amplitude A is governed by the parabolic equation

$$2ik \frac{\partial A}{\partial z} = \nabla_{\perp}^2 A + \frac{\omega^2}{c^2} [\phi(AA^*)] A$$

where

$$\phi = \frac{\omega_{po}^2}{\omega^2} \left\{ 1 - \left[1 + \left(\frac{e}{m_e \omega c} \right)^2 |A|^2 \right]^{-1/2} \right\}. \quad (20)$$

Following Pandey *et al.* 1990, we solve our RSF, Eq. (16) and obtain the extreme value of width of the ripple from

$$\frac{1}{r_1^2} = \left(\frac{\alpha \omega_{po}^2}{4c^2} \right) \left[A_{oo}^2 \left(1 + \frac{\alpha A_o^2}{f^2} \right)^{-3/2} + \frac{1}{r_o^2} \right] \frac{1}{f^2},$$

with a corresponding value of amplification factor (Γ) from

$$\frac{d\Gamma}{dz} = \frac{1}{2k} \left[\left(\frac{\omega_{po}^2}{2c^2} \cdot \frac{\alpha A_{oo}^2}{f^2} \right) \left(1 + \frac{\alpha A_o^2}{f^2} \right)^{-3/2} \right] - \beta(z). \quad (21)$$

2.3. Effect of static magnetic field

When a beam is propagating along the direction of a static magnetic field, two modes of propagation exist namely, extraordinary and ordinary. To see the importance of the magnetic field we analyze the equations of the previous section and see the effect of a uniform external magnetic field on radiation RSF. Consider the propagation of an electromagnetic wave of angular frequency ω_o along the static magnetic field B_o coinciding with the z -axis. The main beam has a Gaussian intensity distribution along its wavefront. In the linear approximation we can assume the electromagnetic wave to propagate in either of the two modes of propagation, the extraordinary or the ordinary mode. The electric vector in the plasma can thus be written as

$$\mathbf{E}_{\pm} = \hat{\mathbf{E}}_{o\pm} \exp[i(\omega_o t - k_{o\pm} z)]$$

where

$$\mathbf{E}_{\pm} = (E_x \pm iE_y), \quad k_{o\pm} = \left(\frac{\omega_o}{c} \right) \varepsilon_{o\pm}^{1/2}.$$

For more details see our earlier paper (Asthana *et al.*, 1998). In the presence of a Gaussian ripple, the total electric field can be written as

$$\mathbf{E}_{\pm} = E_{o\pm} \exp[i(\omega_o t - k_{o\pm} z)] + E_{1\pm} \exp[i(\omega_o t - k_{o\pm} z)], \quad (22)$$

where $E_{1\pm}$ is the electric vector of the ripple. Analyzing on similar lines the appropriate components of the effective di-

electric tensor in a magnetoplasma due to relativistic variation of mass can be written as

$$\varepsilon_{\pm} = \varepsilon_{o\pm} + \varepsilon_{2\pm} \quad \text{and} \quad \varepsilon_{zz} = \varepsilon_{ozz} + \varepsilon_2 \quad (23)$$

where

$$\varepsilon_{o\pm} = \left[1 - \frac{\omega_p^2}{\omega_o^2 \left(1 \mp \frac{\omega_c}{\omega_o} \right)} \right],$$

$$\omega_p^2 = \left(\frac{4\pi n e^2}{\gamma m_e} \right) \quad \text{and} \quad \gamma = \left\{ 1 + \frac{\alpha E_T^2}{\left[1 \mp \frac{\omega_c}{\omega} \frac{1}{\sqrt{1 + \alpha E_T^2}} \right]^2} \right\}^{1/2}$$

is the relativistic factor in presence of magnetic field, with $E_T^2 = (E_{1\pm} E_{o\pm}^* + E_o E_{1\pm}^*)$ and $\omega_c [= (eB_o/mc)]$ is the electron cyclotron frequency. In Eq. (23) the nonlinearity is given by

$$\varepsilon_{2\pm} = \frac{\omega_p^2}{\omega^2 \left(1 \mp \frac{\omega_c}{\omega} \right)}$$

$$\times \left\{ 1 - \left[1 + \frac{\alpha E_T^2}{\left[1 \mp \frac{\omega_c}{\omega} \frac{1}{\sqrt{1 + \alpha E_T^2}} \right]^2} \right]^{-1/2} \right\},$$

$$\varepsilon_{ozz} = \left(1 - \frac{\omega_p^2}{\omega^2} \right),$$

and

$$\varepsilon_2 = \frac{\omega_p^2}{\omega^2} \{ 1 - [1 + \alpha E_T^2]^{-1/2} \}. \quad (24)$$

In similar lines of Section 2.2, we get the RSF equation in presence of external magnetic field as

$$\frac{d^2 f_{\pm}}{dz^2} = \left[\frac{1}{4} \left(1 + \frac{\varepsilon_{o\pm}}{\varepsilon_{ozz}} \right)^2 \frac{1}{R_{d1}^2 f_{\pm}^3} - \frac{\left(1 + \frac{\varepsilon_{o\pm}}{\varepsilon_{ozz}} \right)}{2R_{n1}^2 f_{\pm}^3} \right] \quad (25)$$

where,

$$R_{d1} = k_{\pm} r_o^2; \quad R_{n1} = r_o \left(\frac{\varepsilon_{o\pm}}{\varepsilon_{2\pm}} E_T^2 \right)^{1/2}.$$

3. RESULTS AND DISCUSSIONS

In the preceding sections we have analytically presented the RSF problem of rippled laser beams with plasmas. Based on our earlier work (Asthana *et al.*, 1994) we find that RSF of rippled laser beam can be analyzed like RSF of a Gaussian laser beam in plasmas (see Eq. 9). The saturating nature of a

nonlinear (NL) dielectric constant leads to two values of the critical power: P_{cr1} and P_{cr2} , and the medium behaves as an oscillatory wave guide in the regime $P_{cr1} < P < P_{cr2}$. The ripple gets focused when the initial power of the ripple is greater than the critical power. The focusing is found to be considerably affected by the power of the main beam and the phase angle between the electric vectors of the main beam and the ripple. Comparing the growth of the ripple (Eq. 21) with an earlier work (Pandey *et al.*, 1990) for low intensities, it is seen that a small ripple on the axis of the main beam grows very rapidly with distance of propagation as compared to self-focusing of the main beam. The relativistic intensities with saturation effects of nonlinearity further allows the nonlinear refractive index in the paraxial regime to have slower radial dependence, and thus the ripple extracts relatively less energy from its neighborhood (for more details see Asthana *et al.*, 1999).

Further, in all real situations of relativistic laser–plasma interaction there are always some self-generated magnetic fields which as a matter of fact changes the propagation characteristics of the medium. In Section 2.4, we have presented RSF equation in presence of external static magnetic field. A more detailed investigation of the same is in progress and will soon be reported in our future work. We conclude from the present analysis that focusing conditions depend on the power of main beam, phase angle between electric vectors of main beam and the ripple and also the strength of magnetic field.

ACKNOWLEDGMENTS

One of the authors, Meenu V. Asthana, undertook this work with the support of “THE ABDUS SALAM International Centre for Theoretical Physics,” program for training and research in Italian laboratories, Trieste, ITALY.

REFERENCES

- ASTHANA, M., SODHA, M.S. & MAHESWAR, K.P. (1994). *Journal of Plasma Physics* **51**, 155.
- ASTHANA, M., DESAI, T. & PANT, H.C. (1998). *Physica Scripta Vol. T75*, 267.
- ASTHANA, M.V., GIULIETTI, A., VARSHNEY, D. & SODHA, M.S. (1999). *Journal of Plasma Physics* **62**, 389.
- BORISOV, A.B., SHIRYAEV, O.B., MCPHERSON, A., BOVER, K. & RHODES, C.K. (1995). *Plasma Phys. Controlled Fusion* **37**, 569.
- KOTHARI, N.C. & KOBAYASHI, T. (1983). *Phys. Rev. Lett.* **50**, 160.
- MAIN, P. & MOUROU, G. (1988). *Opt. Lett.* **13**, 467.
- PANDEY, H.D., TRIPATHI, V.K. & SODHA, M.S. (1990). *Phys. of Fluids* **B2(6)**, 1221.
- PATTERSON, F.G., GONZALES, R. & PERRY, M.D. (1991). *Opt. Lett.* **16**, 1107.
- SODHA, M.S., GHATAK, A.K. & TRIPATHI, V.K. (1976). *Progress in Optics* **13**, 171.
- TAJIMA, T. & DAWSON, J.M. (1979). *Phys. Rev. Lett.* **43**, 267.
- TABAK, M., HAMMER, J., GLINSKY, M.E., KRUEER, W.L., WILKS, S.C., WOODWORTH, J., CAMPBELL, E.M. & PERRY, M.D. (1994). *Phys. of Plasmas* **1**, 1626.