LETTERS

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Spectroscopy of laser-plasma accelerated electrons: A novel concept based on Thomson scattering

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The spectrum of relativistic electron bunches with large energy dispersion, like the ones usually generated with laser-plasma acceleration processes, is difficult to obtain with conventional methods. A novel spectroscopic concept, based on the analysis of the photons generated by Thomson scattering of a probe laser pulse by the electron bunch, is presented. The feasibility of a single-pulse spectrometer, using an energy-calibrated charge coupled device as detector, is investigated. Numerical simulations performed in conditions typical of a real experiment show the effectiveness and accuracy of the new method. [DOI: 10.1063/1.1559992]

Thomson scattering of intense laser beams from charged particles has attracted a great deal of interest for a variety of research areas, including electron bunch characterization,[1–3] laser-plasma interactions,[4–7] plasma fusion,[8–10] x-ray generation,[11,12] and medical diagnostics.[13] In this letter we investigate the possibilty of using linear Thomson scattering of a laser pulse impinging at arbitrary incidence angle onto a relativistic electron bunch to design a novel electron energy spectrometer. The basic idea is to measure the spectral and angular distribution of the scattered photons and use it to infer the energy spectrum of the electrons. The concept can be in principle applied to a wide class of electron bunches, but it seems particularly attractive for the study of laser-plasma accelerated electrons.

Electron spectroscopy is currently performed with magnetic spectrometers (see, e.g., Ref. 14), whose use is generally limited to rather well-collimated electron beams, with a moderate energy spread around a roughly known mean value, with some exceptions. The magnetic electron spectroscopy is much more difficult to apply to electron bunches having a broad band energy spectrum and whose mean energy is not well predictable. Such conditions are typical, for example, of electron bunches produced by laser acceleration of plasma electrons (see, e.g., Refs. 15 and 16). In the latter case, when magnetic spectrometers are used, the electron spectrum is usually obtained with several laser shots, each one devoted to produce a portion of the whole spectrum, which results in a poor accuracy. Some of these limitations can be overcome by using suitable magnet configurations, such as the so-called split-pole spectrograph.[17,18] Nevertheless, as the research on laser acceleration of electrons is rapidly growing, there is a need for a new class of spectrometers.

Let us consider a relativistic electron bunch moving in the "Laboratory" frame along the z direction and a laser pulse of Gaussian longitudinal and transverse envelopes of duration T and waist w, intensity I_0, and reduced vector potential of nonrelativistic amplitude $a_0=8.5\times10^{-10}\lambda[\mu\mathrm{m}]\sqrt{I_0}[\mathrm{W/cm}^2]\ll1$. The laser pulse propagates in the $x-z$ plane with angle α^L with respect to z and intersects the electrons trajectories, thus making them oscillating and irradiating (linear Thomson scattering).

The computation of the Thomson scattered photon yield by a bunch of charged particles is based on a classical electrodynamic approach (see, e.g., Ref. 19) and, in the case of *incoherent* scattering, it can be expressed as a summation over the single electrons spectra (as reported, e.g., in Ref. 11).

In order to obtain a closed expression of the photon yield generated by Thomson scattering of the laser pulse by the electron bunch, we will make the following assumptions.

First, the Rayleigh length of the probe pulse is much longer than the bunch longitudinal size so that the bunch interacts with a plane wave laser pulse. This assumption strongly simplifies the computation of the distribution of the scattered radiation $d^2N/d\omega dO|_{\mathrm{single}}$ produced by a single electron, whose general expression can be found in Refs. 20

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and 21, where ω is the angular frequency of the scattered radiation, θ is the scattering angle in a spherical coordinate system, $\mathbf{n}(\theta,\phi)$ is the scattering direction, and $dO \equiv d(\cos\theta)d\phi$ is the solid angle of photon detection.

Second, during the scattering process, each electron oscillates for a very large number of cycles N_0, so that the spectrum of the radiation scattered at a certain angle by the electron is nearly monochromatic. This assumption imposes at least a lower limit to the laser pulse duration T.

Third, the angular distribution of the bunch, characterized by a collimation angle $\Delta\theta_e$ is known. The knowledge of the angular distribution of the bunch, with an accuracy depending on the energy range of interest (see the following), is crucial for the determination of its energy distribution. Possible experimental methods to evaluate the angular distributions in the case of reproducible shots are reported in Refs. 2 and 15, while in nonreproducible bunch configurations a suitable collimator should be introduced.

Fourth, each population of electrons having the same energy has a known density distribution characterized by (energy dependent) longitudinal and transversal bunch sizes σ_L,σ_T, respectively, and position of the center of the spatial distribution at the time $t=0$ denoted with ζ. We stress, however, that *the knowledge of the spatial distribution is not needed in the special case of backscattering* ($\alpha^L=\pi$) *of a probe pulse with suitable spatial parameters*, as shown in the following.

The fifth and last assumption is that the distribution function of the electron bunch is "frozen" during the interaction with the pulse and can be parametrized as $f(\mathbf{x},\mathbf{p},t) \equiv F(\gamma)\Theta_\gamma(\theta)n_\gamma(\mathbf{r}-\beta ct)$, where $F(\gamma)$ is the energy distribution function, Θ_γ and n_γ describe the angular and spatial distribution of electrons having relativistic factor γ, respectively. This assumption is essential if we need that the electron bunch should not be modified *by* the phase of probing and it strongly simplifies the computation of the scattered radiation distribution. In order for assumption to be fulfilled, the effects of the bunch divergence, space–charge, and ponderomotive forces must be negligible during the interaction of the bunch with the probe pulse, i.e., the following conditions must hold:

$$\begin{aligned} &\delta n \approx cT\beta_\perp(\boldsymbol{\nabla}_\perp n) \ll n,\\ &\delta(\beta\gamma) \approx cTN_e r_0/(\sigma_L,\sigma_T)^2 \ll (\beta\gamma),\\ &\delta(\beta\gamma) \approx cT/\gamma(\boldsymbol{\nabla} a_0^2) \ll (\beta\gamma), \end{aligned} \tag{1}$$

n being the electron bunch density distribution and $\boldsymbol{\nabla}_\perp$ and $\beta_\perp$ are the component of the spatial derivative and speed in a direction perpendicular to the bunch propagation z, respectively. Equation (1) implies that

$$\begin{aligned} &\Delta\theta_e cT/\sigma_T \ll 1,\ \ (N_e/\gamma)[cTr_0/\min(\sigma_L,\sigma_T)^2] \ll 1,\\ &(a_0^2/\gamma^2)[cT/\min(cT,w)] \ll 1, \end{aligned} \tag{2}$$

r_0 being the electron classical radius and w the probe waist size.

To evaluate the scattered photon yield with the above-reported assumptions, we start considering the scattered distribution produced by a single electron and making the approximation of its full monochromaticity ($\omega_0 T\to\infty$). Next, for each energy slice a spatial and angular integration of the electrons yields is performed and finally the energy integration leads to

$$\begin{aligned} \frac{d^2N}{d\omega d\cos\theta}(\omega,\theta) = {}& \alpha\frac{\sqrt{2\pi}}{8}a_0^2\int d\gamma\, d\theta_e \Theta_\gamma(\theta_e)N_0(\gamma)\\ &\times F(\gamma)\frac{(\mathcal{H}(\gamma)e^{-\mathcal{K}(\gamma)(\Delta T-\zeta/\beta(\gamma))^2})}{\gamma^2(1-\beta(\gamma)\cos(\theta-\theta_e)^2)}\\ &\times\delta(\omega-\tilde{\omega}(\theta-\theta_e,\gamma)), \end{aligned} \tag{3}$$

where the "resonant" scattered radiation angular frequency $\tilde{\omega}(\theta,\gamma)$ is given by

$$\tilde{\omega}(\theta,\gamma) = \Omega[(1-\beta(\gamma)\cos\alpha^L)/(1-\beta(\gamma)\cos\theta)],$$

Ω being the pulsation of laser pulse. The envelope functions $\mathcal{H}$, $\mathcal{K}$ take into account the spatial integration of the single electron photon yield and in the case of *Gaussian spatial distributions*[2] can be expressed as

$$\begin{aligned} \mathcal{H}^{-2} = {}& \left(1+2\frac{\sigma_T^2}{w^2}\right)\left(1+2\frac{\tilde{T}^2}{T^2}\frac{\sigma_L^2\gamma^2\cos^2\delta}{w^2}\right)\\ &\times\left(1+2\frac{\tilde{T}^2}{T^2}\frac{\sigma_T^2\sin^2\delta}{w^2+2\gamma^2\sigma_L^2(\tilde{T}^2/T)^2\cos^2\delta}\right), \end{aligned} \tag{4}$$

$$\mathcal{K} = 2\frac{\tilde{T}^2}{T^2}\frac{(\gamma\beta c\cos\delta)^2}{w^2+2(\tilde{T}^2/T^2)(\gamma^2\sigma_L^2\cos^2\delta+\sigma_T^2\sin^2\delta)},$$

with $\cos\delta = \sin\alpha^L/\gamma(1-\beta\cos\alpha^L)$ and $\tilde{T}\equiv N_0/\Omega = T\cdot w/(w^2+(\gamma\beta\cos\delta\ cT)^2)^{1/2}$. Expressing the Dirac function in Eq. (3) in terms of the relativistic γ factor of the scattering electron(s), we obtain a closed expression *which links the scattered photon yield spectrum to the energy distribution function of the electron bunch* $F(\gamma)$:

$$\begin{aligned} \frac{d^2N}{d\omega d\cos\theta}(\omega,\theta) = {}& \alpha\frac{\sqrt{2\pi}}{8}a_0^2\int d\theta_e\Theta_{\tilde{\gamma}}(\theta_e)N_0(\tilde{\gamma})F(\tilde{\gamma})\\ &\times\frac{\tilde{\gamma}\mathcal{H}(\tilde{\gamma})e^{-\mathcal{K}(\tilde{\gamma})(\Delta T-\zeta/(\beta(\tilde{\gamma}c)))^2}}{\Omega(\cos(\theta-\theta_e)-\cos\alpha^L)}, \end{aligned} \tag{5}$$

being

$$\tilde{\gamma} \equiv \frac{(\omega\cos(\theta-\theta_e)-\Omega\cos\alpha^L)}{\sqrt{(\omega\cos(\theta-\theta_e)-\Omega\cos\alpha^L)^2-(\omega-\Omega)^2}} \tag{6}$$

the relativistic factor of the electron(s) which generates the scattered photon(s).

We are now able to build up the formula to be used for the energy spectrometer. Let

$$S(E,\theta) \equiv (dN/dEd\cos\theta)|_{\text{exper}}$$

be the detected spectrum of the photon yield corrected by the detection efficiency $\eta(E)$, E and θ being the energy and scattering angle. The energy spectrum of the electron bunch $F(\gamma)$ can be retrieved from $S(E,\theta)$ as

$$F(\gamma)=[8/\sqrt{2\pi}\alpha a_0^2 N_0(\gamma)]E^{\text{laser}}(1-\cos\alpha^L)$$
$$\times[e^{\mathcal{K}(\gamma)(\Delta T-\zeta/(\beta(\gamma)c)^2}/\gamma\mathcal{H}(\gamma)]\mathcal{D}(S,\Theta,\gamma), \qquad (7)$$

where $\mathcal{D}(S,\Theta,\gamma)$ means the *deconvolution* of $S(E,\theta)$ with the known angular distribution function $\Theta_\gamma(\theta_e)$, $E=E^{\text{laser}}(1-\beta(\gamma)\cos\alpha)/(1-\beta(\gamma)\cos(\theta-\theta_e))$ and $E^{\text{laser}}=\hbar\Omega$ is the energy of the incoming photons. The expression (7) is the basic equation of the spectrometer.

Formulas (4)–(7) strongly simplify in the case of backscattering of a probe pulse whose waist w satisfies the condition $w\gg\sigma_T$, i.e., when the laser spot is much larger than the electron bunch transverse size. In this case the laser pulse contains the whole electron bunch provided that the length of the bunch σ_L does not exceed the probe Reyleigh length $Z_R\approx\pi w^2/\lambda$, so the envelope functions $\mathcal{H}$, $\mathcal{K}$ do not depend on the actual shape of the spatial distribution, as considered earlier, and reduce to

$$\mathcal{H}\rightarrow 1, \quad \mathcal{K}\rightarrow 0. \qquad (8)$$

In this case the electron spectrum can be derived from the photon spectrum using the simple relations:

$$F(\gamma)=2.4\times10^{-2}(w^2E^{\text{laser}}/\mathcal{E}\lambda)\mathcal{D}(S(E),\Theta,\gamma),$$
$$E=2E^{\text{laser}}[1/(1-\beta(\gamma)\cos(\theta-\theta_e))], \qquad (9)$$

where $\mathcal{E}$ is the energy delivered by the probe pulse (in Joules), w and λ are in microns, E^{laser} is in electron volts, and we have assumed $\theta\ll1$ and $\gamma\gg1$.

In order to present a full simulation of a possible experimental setup, we will focus on the measure of the spectrum of a relativistic electron bunch produced by laser-plasma acceleration (say laser wake field acceleration, LWFA, or self-modulated-LWFA, see for example Refs. 16 and 22). In this framework, an ultraintense laser pulse inpinges onto a plasma and the strong electric fields of the wake accelerate a large number of electrons at energies exceeding tens of mega electron volts.[15]

To face with a realistic electron bunch, we simulated $N_e=6\times10^8$ electrons with Gaussian angular distribution $\Theta_\gamma(\theta_e)$ (as reported in Ref. 15) having divergence ranging from 20 mrad at low energies to 10 mrad at large energies. The spatial distribution has transverse and longitudinal size $\sigma_T=20\ \mu$m, $\sigma_L=50\ \mu$m, respectively. Finally, the energy distribution $F(\gamma)$ is composed both by an exponentially decreasing component, such as that found in a real experiment[15] and a few mega electron volts full width at half maximum Gaussian peak, taking into account the portion of the bunch (possibly) generated with controlled trapping.[2,23] The probe pulse, which propagates against the simulated electron bunch ($\alpha^L=\pi$), is 0.5 ps long and has wavelength $\lambda=1.053\ \mu$m, energy $\mathcal{E}=0.1$ J, and waist size $w=50\ \mu$m ($a_0\approx0.05\ll1$). Incidentally, we note that these parameters enable us to fulfill the assumptions concerning the pulse duration as well as the constraint on the Rayleigh length with respect to the bunch longitudinal size ($Z_R\approx1$ cm). Also we stress here the fact that both the bunch and the probe parameters fit the requirements for a frozen dynamics of the bunch (for $\gamma>5$) during its interaction with the probe [see Eq. (2)]:

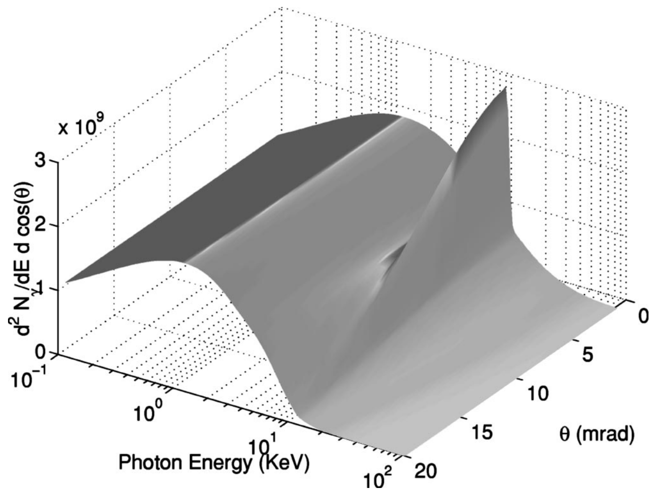

FIG. 1. Numerical simulation of the scattered photon yield integrated onto the azimuthal angle ϕ.

$$\Delta\theta_e cT/\sigma_T\approx5\times10^{-2}\ll1, \quad (N_e/\gamma)(cTr_0/\sigma_T^2)\approx0.1\ll1,$$
$$(a_0^2/\gamma^2)(cT/w)\approx10^{-4}\ll1.$$

The simulation of the detected photon yield is performed with a Monte Carlo method by using Eq. (3), assuming an acceptance angle of the photon detector (a CCD, or charge coupled device, camera, see below) of 30 mrad and a detection efficiency in the range 0.2–1.[26] The angular and spectral distribution of the $\approx10^6$ detected photons is shown in Fig. 1. The data yield is organized in about one thousand channels (one hundred channels in energy and ten in azimuthal angle), resulting in a mean of $\approx10^3$ counts per channel.

The probe and bunch parameters enable the simplified equations (8) and (9) to be used; the result of the analysis, having assumed a detailed knowledge of the angular distribution, is shown in Fig. 2. It is worthwhile to point out that the estimated energy spectrum well reproduces the simulated one also at large energies, where the effect of the bunch divergence is significant. We also note that the results of the analysis of the simulated data clearly show that the electron

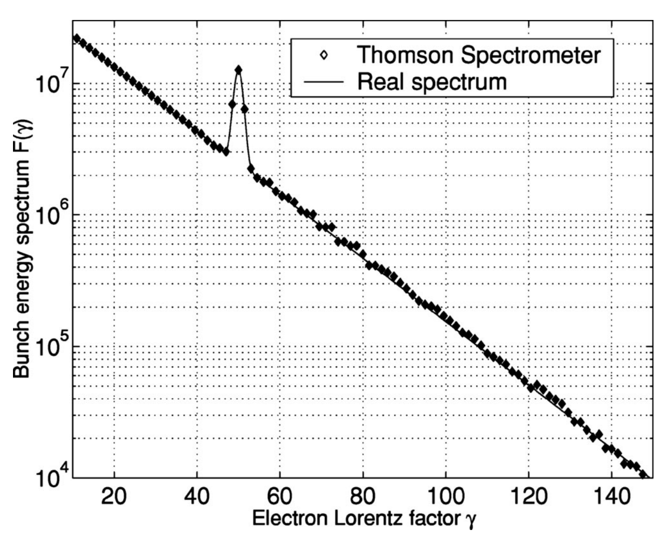

FIG. 2. Comparison between the bunch energy spectrum estimated with the Thomson spectrometer and the simulated spectrum (full line).

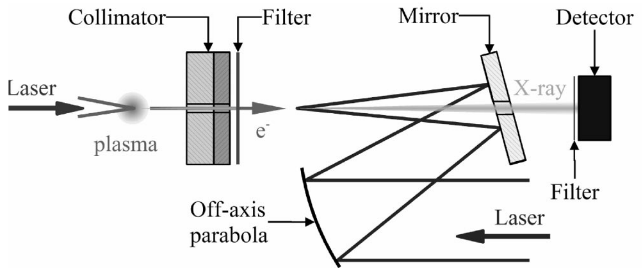

FIG. 3. Example of a possible experimental setup. The probe pulse is focused onto the electron bunch with an off-axis parabola and the scattered photons are detected with an ADC-calibrated CCD camera.

spectrometer is capable of measuring, in a single shot, the very wide energy spectrum with a resolution which is good enough to detect a narrow peak.

A possible experimental setup of the spectrometer is shown in Fig. 3. The probe pulse is focused with an off-axis parabola and directed with a mirror toward the electron bunch. If the electron bunches have angular distributions varying from shot to shot, a way to control the angular divergence of the beam could be the introduction of a suitable collimator, whose aperture θ_{coll} depends on the maximum detected energy. Since $\delta\gamma/\gamma \approx 1/8(\gamma\theta_{\mathrm{coll}})^2$ at large energies, the aperture of the collimator should be limited to $\theta_{\mathrm{coll}} < \gamma_{\mathrm{max}}$. With our setup, with a maximum energy $E_{\mathrm{max}} = 1/2\gamma_{\mathrm{max}} \approx 75$ MeV to be detected, a collimator of aperture $\theta_{\mathrm{coll}} = 5$ mrad would limit the uncertainty on the energy distribution at high energies up to 10%. The Thomson scattered radiation is detected with an energy calibrated CCD camera working in a single-photon regime.[24,25] In the case of laser-plasma produced electrons, nonlinear Thomson scattering of the main pulse with adimensionalized amplitude $a_0^{\mathrm{main}} \approx 10$ and the plasma produces radiation having an energy extending up to ≈ 1 keV.[20] In this case, an aluminum plate 10 μm thick, which filters out the radiation without modifying the ultrarelativistic component of the electron bunch ($E > 2$ MeV), can be inserted. In order to ensure the acquisition of a large number of photons for each shot, the CCD camera should have both a large number of pixels and a reasonable high quantum efficiency at large energies (as, for example, in Ref. 26). Finally, the electron bunch must be deviated, e.g., with a bending magnet (a few cm, 1 T magnet, should be sufficient with the energies considered), to avoid the appearance of noise produced by the electrons impinging onto the CCD.

The study of the uncertainty on the bunch energy spectrum which can be obtained by using such a spectrometer is encouraging, especially in the case of backscattering, where the errors on $\mathcal{H}$ and $\mathcal{K}$ could be made negligible. In this case, with reasonable values of uncertainty on the detected photons energy δE and bunch divergence $\delta(\Delta\theta_e)$ of 100 eV and 1 mrad with reproducible bunches,[2,15] we obtain an estimation of the energy spectrum with uncertainty of a few percent in the very wide energy band 5–500 MeV, provided that the statistics of the photon counts is rich enough (as in the present case).

To summarize, we have shown that linear Thomson scattering of a laser beam onto a relativistic electron bunch can actually be used to retrieve the energy distribution of the electron bunch by the angular resolved spectrum of the scattered photons. In the case of backscattering and when the probe waist size is much higher than the transverse size of the electron beam, the derivation of the electron spectrum strongly simplifies and a very accurate spectrometer can be implemented, which enables single-shot spectra of broadband electron bunches to be retrieved. Calibrated CCD cameras operating in single-photon regime seems to be the ideal photon detector for such a spectrometer.

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