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Nonlinear Propagation of Intense Short Pulses Through Underdense Plasmas

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Abstract

Currently there is much interest in the interaction of high-intensity ultra-short laser pulses with plasmas. Applications include the recently proposed Fast Ignitor Concept for Inertial Confinement Fusion. In the present work we make an analytical investigation of nonlinear propagation of intense short pulses through underdense plasmas. When a laser beam is focused into a plasma, self-focusing and self-channeling can occur as a result of relativistic modification of electron mass in the laser field and the reduction of electron density on the focal region due to the expulsion of electrons by laser ponderomotive force. The paper presents a paraxial theory of self-focusing of intense laser pulses due to expulsion of plasma electrons produced by the extreme ponderomotive force of a focused laser pulse. The nonlinear dielectric constant, self-focusing equation relating the variation of beamwidth parameter with distance of propagation, self-trapping condition and critical power are evaluated. The results suggest a self-focusing of the laser pulse in the plasma.

1. Introduction

The propagation of an ultra-short intense laser pulse through an underdense plasma is an important area of research. The propagation of the laser pulse to an intense focus in a plasma has specific applications in Inertial Confinement Fusion (ICF) in the case of Hohlraum entrance holes and the recently proposed Fast Ignitor Concept (FIC) [1]. With the progress of compact short-pulse multitera-watt laser systems [2], it has become possible to produce MeV electrons using preformed plasmas [3] or pulsed gas jets [4] produced from solid targets [5]. The fast ignitor concept, relevant to inertial confinement fusion, enhances the interest in this process as well as in laser propagation and channel formation. In an underdense plasma, electrons and ions tend to be expelled from the focal volume by the ponderomotive pressure of an intense laser pulse, and the channel so created can act as a propagation guide for the laser beam. Depending on the quality of the laser beam, the cumulative effects of ponderomotive and relativistic self-focusing (RSF) [6] can significantly increase the laser intensity. When a laser beam with an intensity profile peaked on axis is incident into a plasma, self-focusing and self-channeling can occur as a result of two effects: the relativistic modification of electron mass in the laser field and the reduction of the electron density on axis due to the expulsion of electrons by laser ponderomotive force. In ponderomotive self-channeling the laser ponderomotive force expels electrons from the axis (the ions do not move much because of their greater mass) and prevents their return, despite the Coulomb force which arises from charge separation.

Several groups [7–9] claimed to observe plasma density depression channels produced in this way. However, much of the highly nonlinear plasma phenomena which occurs during such interactions are not well understood and experiments which address the fundamental physics of these interactions are required in order to properly evaluate the suitability of these intense laser – produced plasmas for such applications. In view of ongoing investigations we present here an analytical investigation of nonlinear propagation of ultra-short intense laser pulses through underdense plasmas. Based on paraxial theory self-focusing of laser pulses due to expulsion of plasma electrons produced by the extreme ponderomotive force of a focused laser pulse is studied. In Section 2 nonlinear dielectric constant, self-focusing equation relating the variation of beamwidth parameter with distance of propagation, self-trapping condition and critical power are evaluated. Results and discussions are made in Section 3 with scope of future work.

2. Analytical formulation

2.1. *Nonlinear dielectric constant*

If a very intense laser beam penetrates a plasma, RSF of the laser beam arises because of the relativistic velocity of the electrons which enters the relativistic mass of the electrons in the plasma frequency as

$$\omega_p^2 = \frac{4\pi n_e e^2}{\gamma m_e}.$$

Here, m_e is the rest mass of the electron and n_e the electron density. The relativistic Lorentz factor γ depends on the electric field strength E. For example, the factor γ for a circularly polarised wave of frequency ω is given as

$$\gamma = \left[1 + \left(\frac{e}{m_e \omega c}\right)^2 E^2\right]^{\frac{1}{2}}.$$

An important consequence of RSF is the increase of the energy density of the laser field up to extremely high values in the region around the focal point. That means nonlinear forces are developing which drive electrons and ions out of the focus region. For a circularly polarized light the nonlinear force is given by

$$F_{NL} = -\frac{e^2}{2m_e \gamma \omega^2} \nabla E^2 = -\nabla(m_e c^2 \gamma). \quad (1)$$

Following Sodha *et al.* [10], the electron concentration under the assumption that the plasma is unperturbed at large distances from the beam axis is given by

$$N_e = N_o \exp\left(-\frac{3m\alpha}{4M\gamma}EE^*\right) \quad \text{where} \quad \alpha = \left(\frac{e^2M}{6m^2\omega^2k_oT_o}\right). \tag{2}$$

The corresponding expression for the dielectric constant is

$$\varepsilon = \varepsilon_o + \phi(EE^*) \tag{3}$$

with

$$\varepsilon_o = \left(1 - \frac{\omega_p^2}{\omega^2}\right).$$

Here ε_o and ϕ represent respectively the linear and nonlinear dielectric constants.

2.2. *Self-focusing equation*

Consider the propagation of a Gaussian laser beam along the z-direction; at $z = 0$ the intensity distribution of the beam is given by

$$EE^* = E_o^2 \exp\left(-\frac{r^2}{r_o^2}\right)$$

where r is the radial coordinate of the cylindrical coordinate system and r_o is the initial beam width.

The wave equation governing the electric vector of the beam in plasmas with dielectric constant given by Eq. (3) can be written as

$$\nabla^2 E + \frac{\omega^2}{c^2}\varepsilon E = 0. \tag{4}$$

In writing Eq. (4), the term $\nabla(\nabla \cdot \boldsymbol{E})$ has been neglected, which is justified when

$$\left(\frac{c^2}{\omega^2}\right)|(\nabla \ln \varepsilon)/\varepsilon| \ll 1.$$

Using the WKB approximation and following Asthana *et al.* [11], we can write

$$E(r,z) = A(r,z)\left[\frac{k(o)}{k(z)}\right]^{\frac{1}{2}} \exp[-i \int k(f)dz] \tag{5}$$

where

$$k(f) = \frac{\omega}{c}[\varepsilon_o'(f)]^{\frac{1}{2}}; \quad k(o)\frac{\omega}{c}[\varepsilon_o'(f=1)]^{\frac{1}{2}}.$$

Substituting for E and ε, from Eqs (5) and (3) into Eq. (4)and separating the real and imaginary parts, we obtain

$$2\frac{\partial S}{\partial z} + \left(\frac{\partial S}{\partial r}\right)^2 + \frac{\omega^2}{c^2}\frac{\varepsilon_1(f)}{k^2(f)}r^2 = \frac{1}{k^2A_o}\left(\frac{\partial^2 A_o}{\partial r^2} + \frac{1}{r}\frac{\partial A_o}{\partial r}\right)$$

and

$$\frac{\partial A_o^2}{\partial z} + \frac{\partial S}{\partial r}\frac{\partial A_o^2}{\partial r} + A_o^2\left(\frac{\partial^2 S}{\partial r^2} + \frac{1}{r}\frac{\partial S}{\partial r}\right) = 0. \tag{6}$$

The solution of the above equations can be written as

$$S = \tfrac{1}{2}r^2\beta(z) + \eta(z), \qquad A_o^2 = \frac{e_o^2}{f^2}\exp\left(-\frac{r^2}{r_o^2 f^2}\right) \tag{7}$$

where

$$\beta = \left(\frac{1}{f}\right)\frac{df}{dz}.$$

It can be seen from this that β represents the inverse radius of curvature of the wavefront while $r_o f$ is the width of the beam. In the geometrical-optics approximation $r = r_o f(z)$ represents a ray in a plane containing the z axis. Substituting for S from Eq. (7) into Eq. (6) using the paraxial ray approximation (i.e., $(r/r_o f)^4 \ll 1$) equating the coefficients of r^2 on both sides of the resulting equation we obtain

$$\frac{d^2 f}{dz^2} = \frac{1}{k^2(f)r_o^4 f^3} + \frac{\omega^2\varepsilon_1(f)f}{c^2k^2(f)} \tag{8}$$

with

$$\varepsilon_1 = -\left(\frac{3m\alpha}{4M\gamma}\right)\left(\frac{\omega_p^2}{\omega^2}\right)\exp\left(-\frac{3m\alpha}{4M\gamma}\left\{\frac{k(o)}{k(f)}\frac{E_o^2}{2f^2}\right\}\right).$$

Equation (8) governs the variation of the beam-width parameter f with distance of propagation.

2.3. *Self-trapping and critical power*

Substituting for E from Eq. (5) and S, A_o and β from Eq. (7) into Eq. (3) and using the paraxial ray approximation, ϕ can be written as

$$\phi(EE^*) \approx \phi\left(\frac{k(o)E_o^2}{k(f)2f^2}\right) - r\left\{\left(\frac{3m\alpha}{4M\gamma}\right)\left(\frac{\omega_p^2}{\omega^2}\right)\exp\left(-\frac{3m\alpha}{4M\gamma}\left[\frac{k(o)E_o^2}{k(f)2f^2}\right]\right)\right\}^2 \tag{9}$$

correct to terms in r^2, where

$$\phi\left(\frac{k(o)E_o^2}{k(f)2f^2}\right) = \frac{\omega_p^2}{\omega^2}\left\{1 - \exp\left(-\frac{3m\alpha}{4M\gamma}\left[\frac{k(o)E_o^2}{k(f)2f^2}\right]\right)\right\}.$$

For an initial plane wave front of the beam the initial conditions on f are $f(z = o) = 1$ and $df/dz|_{z=0} = 0$. When the two terms on the right hand side of Eq. (8) cancel each other at $z = o$, $(d^2f/dz^2) = 0$. Since df/dz is also zero and $f = 1$ at $z = 0$, $f = 1$ for all values of z. In other words, the beam propagates without convergence or divergence and one gets the condition for self-trapping with corresponding critical power of the beam as

$$P = \frac{c}{8\pi}\int_0^\infty \varepsilon^{\frac{1}{2}} E_{ocr}^2 \exp\left(-\frac{r^2}{r_0^2}\right)2\pi r dr = \frac{1}{8}cr_0^2E_{ocr}^2[\varepsilon_o'(f=1)]^{\frac{1}{2}} \tag{10}$$

where

$$\varepsilon_o' = \varepsilon_o + \frac{\omega_p^2}{\omega^2}\left\{1 - \exp\left(-\frac{3m\alpha}{4M\gamma}\left[\frac{k(o)E_o^2}{k(f)2f^2}\right]\right)\right\}.$$

Equation (10) has two critical powers P_{cr1} and P_{cr2}, corresponding to two values of electric field strength, say E_{ocr1} and E_{ocr2} (such that $E_{ocr1} < E_{ocr2}$). The beam can be self-focused when its power P lies between the two critical values, i.e., $P_{cr1} < P < P_{cr2}$.

3. Results and discussions

From the analysis presented in the previous section we find that under WKB and paraxial ray theory relativistic ponderomotive self-focusing can be understood. The saturating nature of the nonlinear dielectric constant leads to two values of the critical power as discussed in an earlier work [11]. On making numerical estimation for typical parameters of laser-plasma interaction relevant to the fast ignitor concept, it is seen that periodic structure is a general feature of beam propagation in a nonlinear medium. This arises from a competition between diffraction and self focusing terms in the wave, Eq. (8). The beam radius oscillates (in a self made oscillatory waveguide) around a value for which diffraction and self-focusing terms exactly cancel each other which signifies self channeling of the laser beam. The critical power is found to be higher in comparison to the critical power due to relativistic variation of mass which as a matter of fact increases the self-focusing effect. The work has significant relevance for channel formation in underdense plasmas which will be reported next.

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